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Methodology to identify the relevant uncertainties

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1 Introduction

Due to the liberalisation of the energy sector the planning of the CHP operation faces much uncertainties. The inclusion of the trading options into a deterministic model is not satisfying the way it has been done so far. Totally new model approaches are thus required to account for the uncertainties the CHP operators are exposed to. In this deliverable we discuss the different kind of uncertainties which are of interest for CHP operation.

For operators of CHP systems many uncertainties exist regarding the input parameters for the unit commitment model and the model for the load dispatch. In addition to the uncertainties about electricity prices at the spot market, electricity demand and fuel prices CHP operators face the uncertainty in heat demand. The various uncertainties are of different importance for different systems. For example for a long term model of a system with a heat storage the uncertainty in heat demand will be less important than for a system without heat storage. A general statement about the importance of the different uncertainties for different systems can not be made. The influence of the uncertainties has to be tested for each system individually. In this report one approach to test the influence of the uncertainty of the different parameters is proposed. However, first the uncertain parameters will be discussed in general.

2 Uncertainties

In order to find out the main uncertainties the operators are exposed to, we asked our industrial partners to classify the uncertainties. We obtained following Table 2.1

Table 2.1: Relevant uncertainties for CHP-operation

Uncertainty	Influencing factors	Type	Short term relevance (2 weeks)	Long term relevance (1 year)
Heat demand	Temperature, wind, seasonality	Quantity risk	medium	high
Electricity demand	Temperature, sun light, wind, seasonality, time of day, weekday	Quantity risk	high	Medium, only when reserve power is considered
Plant outages	unknown	Quantity risk	medium	medium
Fuel price	Economic development	Price risk	Very low	low
Spot price	Seasonality, time of day, weekday	Price risk	Medium	high
Forward price	Seasonality, expectations	Price risk	low	high
Counter party risk	Economic development	other	low	low
Legislative risk		other	low	low

Those uncertainties with high relevance either for the short or long term model will be discussed in detail in the following.

Spot and Forward prices

For both the short-term and long-term models the electricity prices will be the main uncertain factor. They have a big influence on the unit commitment and on the load dispatch. The behaviour of electricity prices differs from the behaviour of other commodity prices. One reason for this is that electricity is a non-storable good, implying that inventories cannot be used to arbitrage prices over time. For example, the behaviour of electricity spot prices of the German LPX spot market can be characterised by the following observations.

Strong mean reversion: Deviations of the price from the equilibrium level due to random perturbations are corrected at a certain rate (see Figure 2.1).

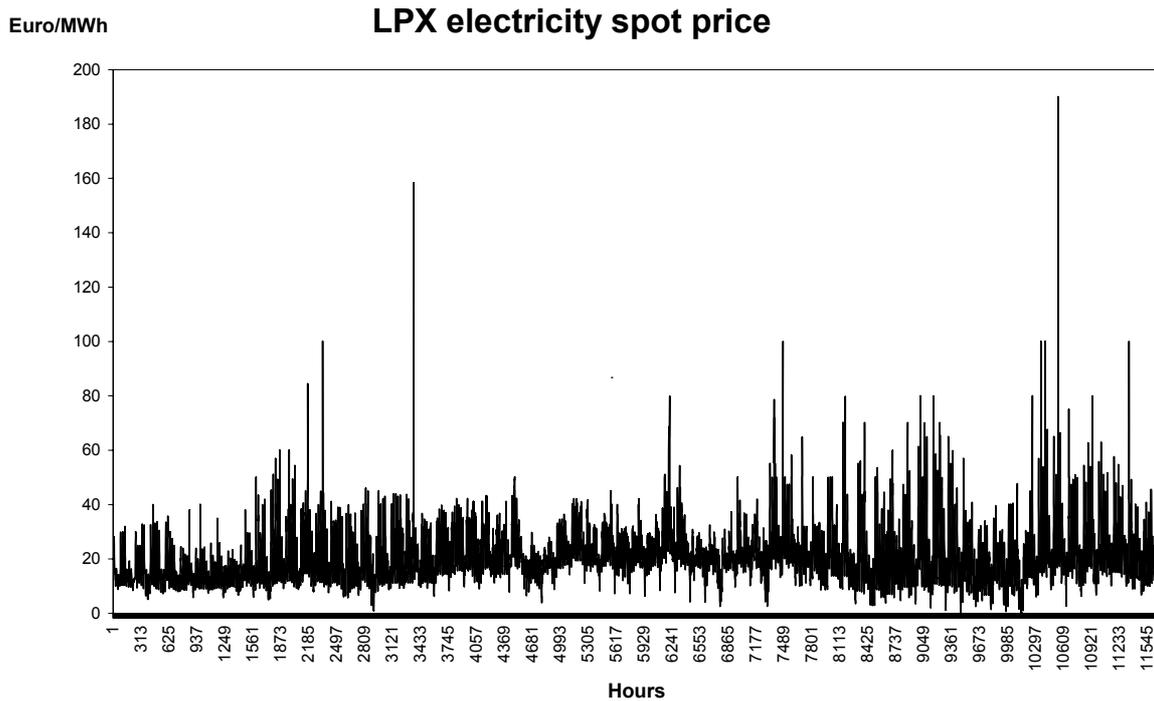


Figure 2.1: LPX Spot Market Prices over the period June 16, 2000 - October 15, 2001

Time of day effect - this is demonstrated in Figure 2.2 where average hourly electricity spot prices measured in Euro per Megawatt hour (Euro/MWh) for weekdays and weekends are presented. As expected, prices are on average higher during the week when demand is higher. The price begins to increase at 5:00 during the workday and continues to increase until 12:00 when there is the first and biggest peak of the day. Then the price begins to fall until 17:00 and after reaching its lowest point it starts to increase again until 18:00 when it reaches the second (and smaller) peak of the day. Prices begin to fall thereafter as the workday ends and demand shifts to primarily residential usage.

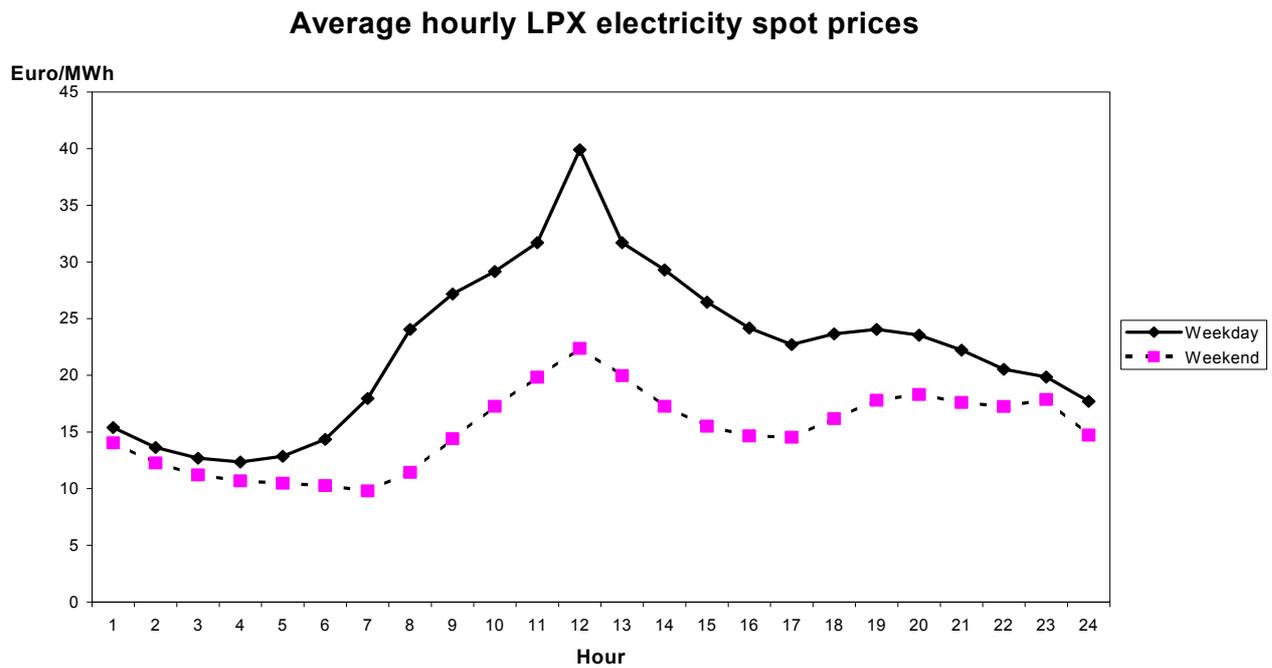


Figure 2.2: Average hourly LPX electricity spot prices across the entire sample

In Figure 2.2 the *Weekend/weekday effect* can be studied, too. Prices distributions deviate from the normal distribution in a significant way (see Figure 2.3).

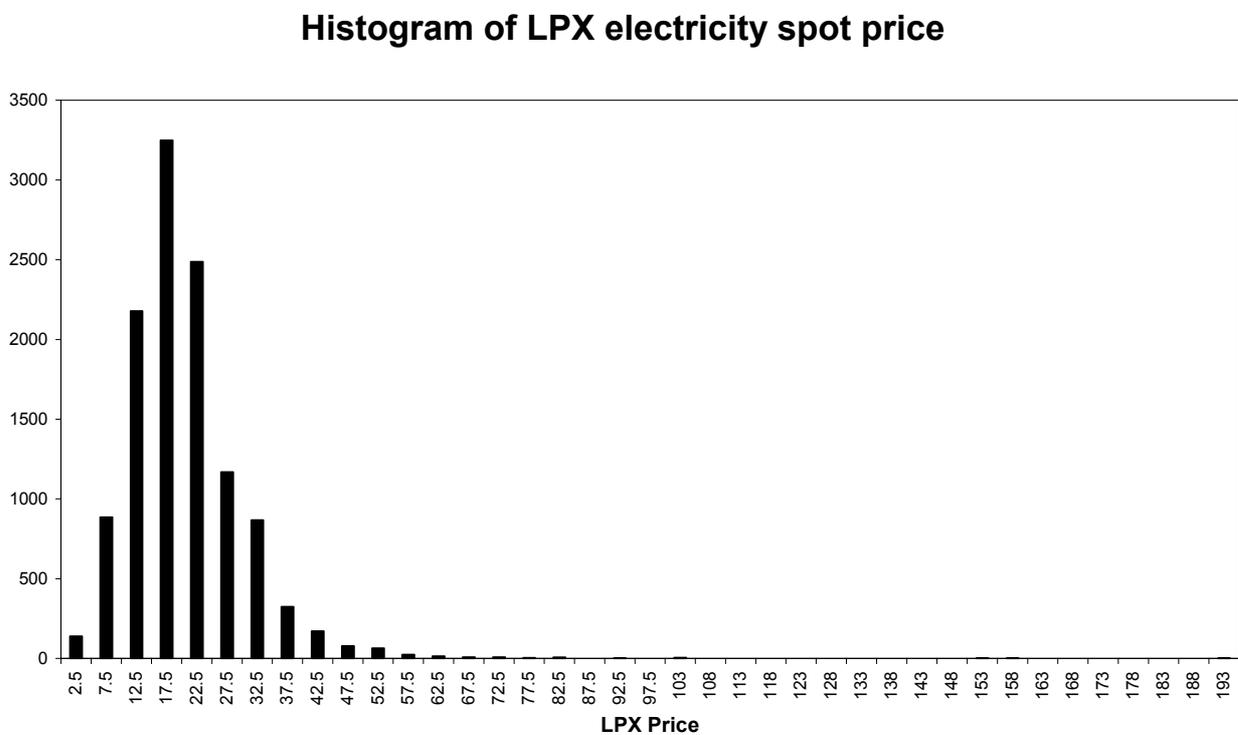


Figure 2.3: Histogram of LPX electricity spot price over the period June 16 -October 15, 2001 (Mean = 20.33, St. Dev. = 9.5, Skewness = 2.37, Kurtosis = 23.5)

Table 2.2 presents summary statistics for the whole sample of hourly electricity prices and for all 24 hours of daily electricity prices. Note that the electricity prices for the first peak hour (12:00) have the biggest mean and standard deviation and the electricity prices for the second peak hour (18:00) have the biggest skewness and kurtosis.

Table 2.2: Descriptive statistics for LPX electricity spot prices over the period June 16, 2000 - October 15, 2001

Hour	Mean	St. Dev.	Skewness	Kurtosis
1	15.01	4.04	0.077	2.960
2	13.25	4.03	-0.038	2.574
3	12.29	4.04	-0.004	2.650
4	11.88	4.08	0.010	2.570
5	12.18	4.23	-0.147	2.620
6	13.19	4.38	-0.434	2.785
7	15.62	5.43	-0.601	2.694
8	20.43	8.11	-0.028	2.734
9	23.52	8.89	0.348	3.500
10	25.75	9.26	0.861	5.359
11	28.30	10.01	1.105	5.623
12	34.87	16.59	2.758	19.950
13	28.34	10.07	3.526	36.815
14	25.84	9.14	1.201	8.464
15	23.31	8.19	0.783	4.687
16	21.44	7.08	0.567	4.435
17	20.37	6.51	0.603	4.247
18	21.52	9.54	6.412	88.920
19	22.25	7.69	1.329	6.311
20	22.04	7.03	1.683	12.333
21	20.90	5.53	1.360	9.290
22	19.60	4.25	1.340	13.200
23	19.29	3.65	0.641	7.140
24	16.86	4.05	-0.135	3.388
Whole sample	20.33	9.50	2.370	23.500

For systems without combined heat and power producing units the plan for the unit commitment can be made on the basis of the spot market prices. Under the assumption of a totally liberalised market and without operational restrictions, in general the unit will run if the spot market price will be above or equal to the variable costs for production. Otherwise the unit will be shut down. For CHP systems, on the other hand, the heat demand also has a great influence on the optimal unit commitment plan. Consequently the heat demand has to be considered. However, the level of the spot market price has an influence whether the unit will run in condensing mode or not. Situations may occur when the unit will even be shut down if the heat demand is low enough.

Heat Demand

Heat demand is very important for both the short term and long term models. If the prediction of the heat demand is inaccurate, it may result in non-optimal unit commitment, which has to be corrected afterwards causing higher costs.

For systems with a heat storage this correction can be made easier than in systems without a storage. For systems without substantial heat storage capacities one has to study the deviations of the heat forecasts. These deviations have to be related to the heat demand, the capacity of the heat storage and the loading and unloading speed.

The following Figure 2.4 shows BEWAG's heat demand during the period July 1, 1999 to May 13, 2001.

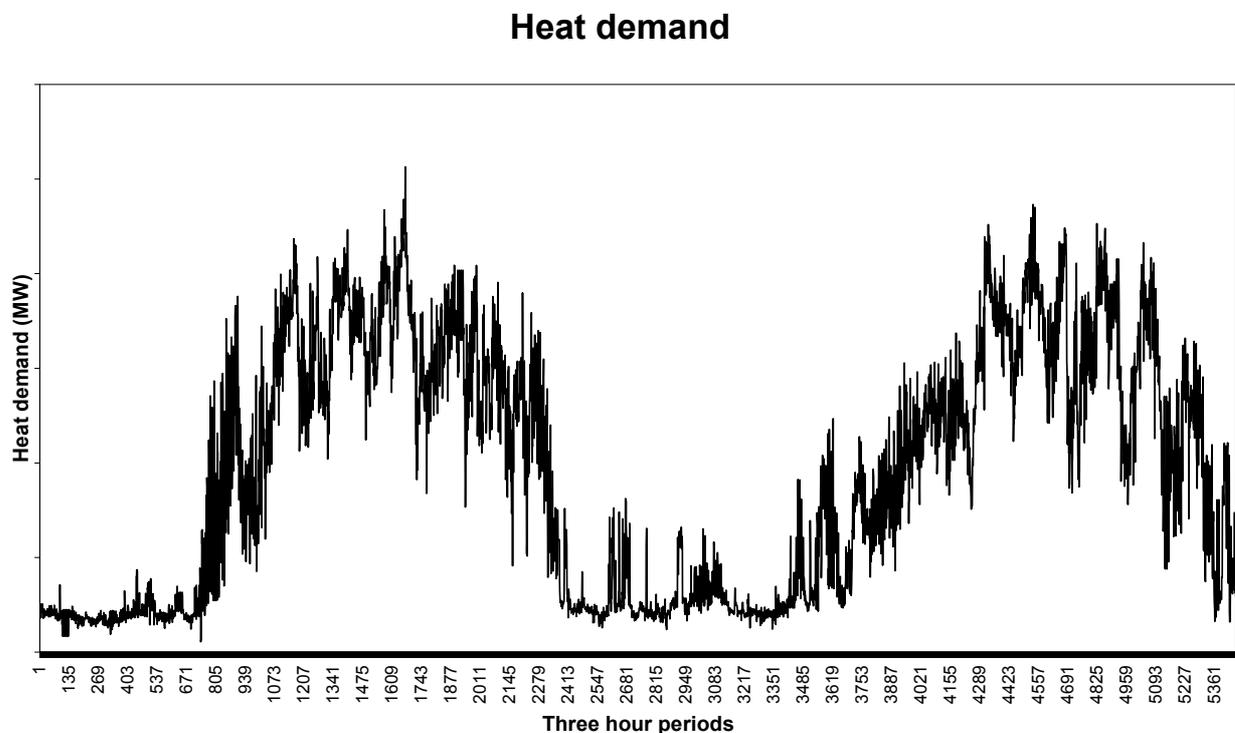


Figure 2.4: Heat demand over the period July 1, 1999 - May 13, 2001

The time series analysis for heat demand shows that heat demand can be characterised by the following effects:

- Time of day effect.
- Weekend/weekday effect.
- Seasonal effects.
- Time varying volatility

Table 2.3 present the summary statistics of the original data and Table 2.4 of the de-seasonalised heat data.

Table 2.3: Descriptive statistics for Heat Demand (MW) over the period July 1, 1999 - May 13, 2001 (original data)

Three hour period	Mean	St. Dev.	Skewness	Kurtosis	Correlation with the temperature
1-3	373.61	259.25	0.155	1.544	-0.948
4-6	377.39	260.09	0.128	1.534	-0.946
7-9	422.69	267.01	-0.015	1.565	-0.947
10-12	407.18	263.87	0.120	1.627	-0.948
13-15	357.69	247.71	0.360	1.734	-0.930
16-18	353.66	251.71	0.364	1.683	-0.933
19-21	373.08	260.31	0.263	1.626	-0.939
22-24	385.63	264.13	0.152	1.572	-0.949
Whole sample	380.95	260.15	0.195	1.597	-0.926

Table 2.4: Descriptive statistics for Heat Demand (MW) over the period July 1, 1999 - May 13, 2001 (de-seasonalised data)

Three hour period	Mean	St. Dev.	Skewness	Kurtosis
1-3	0.58	105.37	0.178	4.217
4-6	0.56	106.84	0.057	3.861
7-9	0.70	112.76	0.015	3.959
10-12	0.80	108.40	0.217	4.049
13-15	0.86	106.33	0.294	4.669
16-18	0.64	102.81	0.369	4.158
19-21	0.61	102.06	0.371	3.884
22-24	0.47	102.99	0.213	4.086
Whole sample	0.00	106.72	0.101	3.631

Electricity load

The uncertainty in electricity load formerly was of big importance for the unit commitment and load dispatch. But under the assumption of a totally liberalised market with the possibility to buy and sell electricity this is no longer true for short and long term planning. For given heat demand, the electricity production only depends on the spot market price under the restrictions of minimal and maximal power.

For systems of ultra short term planning however the uncertainty in the load is still significant, as the operator can not react and buy or sell additional electricity at the spot market in such short times.

3 Stochastic optimisation

The influence of the uncertainties of the different parameters can be included in the optimisation models with the help of stochastic optimisation.

3.1 Theory

Stochastic Optimisation is used when one or several parameters of a model (e.g. prices, demand) are random (that means stochastic) and cannot be forecasted exactly. Knowing the distribution of the parameter(s), one can derive the optimal decisions using stochastic programming. Here, optimal means, that the expected value of the goal function will be maximised.

Deterministic linear models look like

$$\begin{aligned} \max \quad & c^T z \\ \text{s.t.} \quad & Az = b \\ & x \geq 0 \end{aligned}$$

When a parameter is uncertain it is reasonable only to finalise those decisions that cannot be postponed further. Decisions that can wait will be made in the future when one has more information. In the context of stochastic optimisation, the decision z is partitioned into x and y , where x is decided now and y will depend on the future development.

Furthermore, since y depends on the future, y is denoted as y_s where the label s denotes the dependence on the observed future scenario s . If the future scenario s is observed with a probability p_s , then the stochastic model can be written as

$$\begin{aligned} \max \quad & c^T x + \sum_s p_s d_s^T y_s \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \\ & T_s x + W_s y_s = r_s \quad \forall s \in S \\ & y_s \geq 0 \quad \forall s \in S \end{aligned}$$

In literature not only discrete probability distributions are considered, but also continuous distributions. Therefore the vector ξ is introduced to represent random variables

for the uncertain parameters. Further, if ω denotes a possible realisation of the random variable ξ the problem can be reformulated as:

$$\begin{aligned} \text{RP} = & \max_x c^T x + E_{\xi} Q(x, \xi(\omega)) \\ \text{s.t.} & \quad Ax = b \\ & \quad x \geq 0 \end{aligned}$$

with

$$\begin{aligned} Q(x, \xi(\omega)) = & \max_{y(\omega)} d_{\xi}^T y \\ \text{s.t.} & \quad T(\omega)x + Wy(\omega) = r(\omega) \\ & \quad y \geq 0. \end{aligned}$$

This formulation is valid for a two stage model, where information is revealed only once in the future. When information is revealed multiple times, e.g. one can recursively define a multi-stage model. The formulation takes the following form:

$$\begin{aligned} \max_{x^1} & c^1 x^1 + E_{\xi^2} \left[\max_{x^2(\omega^2)} c^2(\omega) x^2(\omega^2) + E_{\xi^3} \left[\max_{x^3(\omega^3)} c^3(\omega) x^3(\omega^3) + \dots + E_{\xi^n} \left[\max_{x^n(\omega^n)} c^n(\omega) x^n(\omega^n) + \dots \right] \dots \right] \right] \\ \text{s.t.} & \quad T^1(\omega)x^1 + W^2 x^2(\omega^2) = r^2(\omega) \\ & \quad T^2(\omega)x^2 + W^3 x^3(\omega^3) = r^3(\omega) \\ & \quad \dots \\ & \quad T^{N-1}(\omega)x^{N-1} + W^N x^N(\omega^N) = r^N(\omega) \end{aligned}$$

3.2 Advantages and disadvantages of stochastic programming

Stochastic optimisation takes into account all the possible realisations of the stochastic parameter(s), in deriving the optimal decision and does not consider the scenarios independently as the approach of scenario esanalysis does (compare Figure 3.1). The model formulation for stochastic programming is rather complex but one can benefit if the distribution of the uncertain parameters can be determined.

It can be shown that the expected value of the goal function of the deterministic model is lower than the expected value of the stochastic optimisation model if all random values are replaced by their expected values. For a maximisation problem this means that the expected profit can be increased by using stochastic optimisation. Furthermore stochastic optimisation allows us to define an additional restriction on the minimum profit which has to be reached independently of the scenario, which will eventuate .

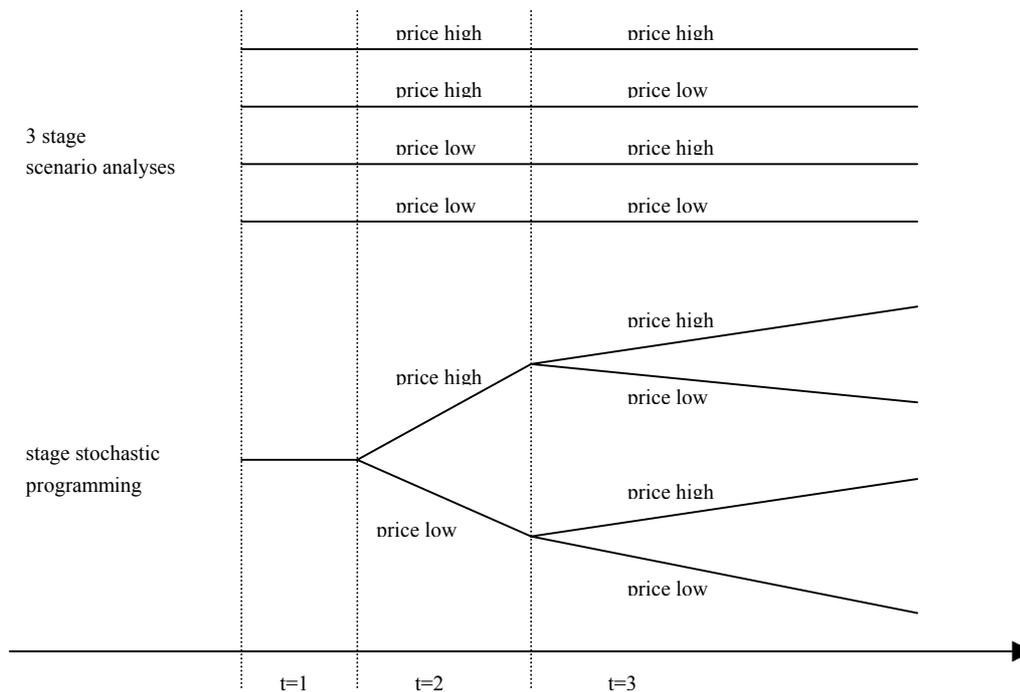


Figure 3.1: Difference between stochastic optimisation and stochastic programming

The optimal decision at the first time step depends on the possible realisation of the uncertain parameter at the following time steps. (e.g. the decision to start-up a power plant at time step 1 depends on the possible realisation of the stochastic LPX-spot market price process at time steps 2 and 3). If the variance of the parameter increases, the optimal decisions will be affected, even if the expected value of this parameter remains the same.

3.3 Scenario creation

Stochastic optimisation models can be extended in a number of ways. One of the most common is to include more stages. With a multistage problem, a decision is taken now, then another decision is taken when some uncertainty has been resolved. The second decision is then based on what has happened. As the model size increases very fast, we have to reduce the number of scenarios to keep computing times acceptable. Different approaches exist, one of them is barycentric approximation, which creates scenarios with the help of barycenters that represent all possible scenarios very well /Frauendorfer 1992/. Another possibility for scenario selection is based on Monte Carlo simulation. Many scenarios will be created and then merged to one scenario tree by scenario reduction methods /Dupacova et al 2000/.

3.4 Value of stochastic optimisation

Most models for unit commitment do not take into account the randomness of the different uncertain parameters. They replace the random parameters by their expected values and solve then the so called expected value problem. Let $\omega = \bar{\xi}$, that means that only one scenario, namely the expected value scenario, will incur. The value of the goal function is called EV and can be written as

$$\begin{aligned}
 EV &= \max_x z(x, \bar{\xi}) = cx + \max_{y(\bar{\xi})} (d(\bar{\xi})y(\bar{\xi})) \\
 \text{s.t. } & Ax = b \\
 & T(\bar{\xi})x + W(\bar{\xi})y(\bar{\xi}) = r(\bar{\xi}) \\
 & x \geq 0 \\
 & y(\bar{\xi}) \geq 0
 \end{aligned}$$

The solution to the problem is denoted as $\bar{x}(\bar{\xi})$. To always take the decision $\bar{x}(\bar{\xi})$ for all scenarios $\omega \in \Omega$ will surely not be the optimal case. The expected value of the goal function when using the solution of the expected value model is defined as

$$EEV = E_{\xi} (z(\bar{x}(\bar{\xi}), \xi))$$

The number EEV measures how the expected value model performs, when the decision $\bar{x}(\bar{\xi})$ is taken on the basis of expected values. The expected goal function of the stochastic programming recourse model is denoted as RP and was defined under 3.1. It is shown in Birge and Loveaux /Birge, Loveaux 1997/ that the following relation is valid:

$$VSS = RP - EEV \geq 0.$$

This means that the goal function's expected value of the stochastic optimisation problem will be better than the expected value of deterministic programming. The important questions for CHP operators is then how big this difference is. Is it worth to use stochastic programming, when only an improvement in the range of promise can be reached? To find out how much stochastic programming is of advantage and which are the most uncertain parameters, a deterministic comparison is required and we propose the following procedure to benchmark stochastic programming against deterministic optimisation.

STEP 1: Formulation of the stochastic optimisation model.

First all equations have to be extended to the stochastic model. Before this can be done, the structure of the scenario tree has to be determined. It is not needed to know the precise scenarios that constitute the tree, but the number of stages and the time steps when a new stage will be introduced have to be known. It is sensible to choose those time steps for the beginning of a new stage, when a decision has to be taken and when new information about the uncertain variables can be obtained.

If new information about the uncertain parameters is revealed at a particular point of time without having the possibility to revise former decisions, then there is no need for a more detailed modelling of this time period by creating an additional stage in the model in order to model the possible outcomes of this uncertain parameter. Thus not the point of time of availability of the new information is crucial, but the point of time of its processing. Such a situation arises for example during night hours when the load demand can be forecasted more precisely. With no personal being at work for changing the state of the units this new information has no value. It is only relevant at the future time point, when a change of the unit states is possible.

STEP 2: Creation of scenario tree

The main algorithms for the construction of the scenario tree consists of two steps. In the first step several scenarios are selected from historical data. Alternatively, if such data is not available, the scenarios are created by Monte Carlo simulations on the basis of the underlying stochastic processes. In a second step these scenarios are merged together to form the so called scenario tree. Algorithms for the merging procedure can be found in /Gröwe-Kuska et. al 1999/.

A different approach is to calculate the conditional distributions of the uncertain parameters. Those distributions are discretised and those values and the probabilities are taken as branches of the trees (see Fig 3.1).

STEP 3 : Solve stochastic model on the basis of the scenario tree

The model is formulated as a stochastic model. If the model is linear then the deterministic equivalent can be built and can be solved with CPLEX /Cplex 2001/. As for integer problems this is very time consuming. In future approaches, using Lagrangian relaxation for stochastic programming problems has to be studied. As results one gets the decisions that have to be taken at the first stage. These decisions takes into account that the parameters for the following time periods are volatile. Under the assumption that exactly the chosen scenarios occur, the solution of the model also deliver the solution for the other time steps.

STEP 4: Deterministic model and rolling planning

In order to compare the results with the stochastic model, one has to allow rolling planning for the deterministic case. In the stochastic model it is assumed that additional information about the uncertain future is revealed at each stage (e.g. at the end of each day, the realisation of the random parameters like heat demand and electricity prices for the next day) This additional information can be used to improve the forecast of the expected value/distribution of the uncertain parameters of the following day. Therefore one has to allow rolling planning and the possibility to use the actual information in the deterministic model. For each stage of the stochastic programming tree and each branch at the considered stage the corresponding expected value models have to be solved. On the basis of the solutions of these models the value EEV (expected value of expected value model) can be determined. From the stochastic optimisation we know the value of the goal function of the whole optimisation problem so that we can calculate the value of the stochastic solution. The bigger this value is the bigger are the benefits of stochastic modelling.

3.5 Practical application

A small demo model was set up to calculate the value of stochastic optimisation /Brand et al. 2001/. The optimal unit commitment and the economic load dispatch were determined for a turbine which produces electricity that can be sold at the LPX-Spot market with the restriction of a take-or-pay contract for the used fuel. The prices at the LPX-Spot market were assumed

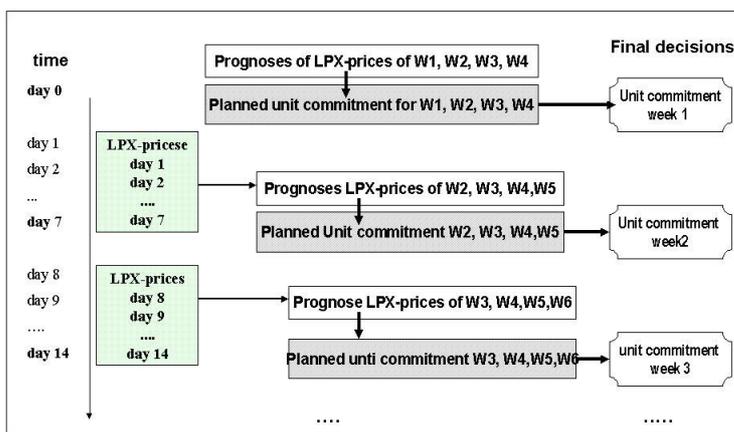


Figure 3.2: Decision process for unit commitment

to be stochastic and to follow a mean-reversion process. Each week, the operator has to announce how much fuel the turbine will need. At the beginning of the first week he/she has to determine roughly the unit commitment of the first week. To calculate the optimal commitment for the first week, he/she takes into account the following 4 weeks, as the fuel is restricted

over those four weeks. The result of the optimisation is a planned unit commitment for the weeks W1,W2,W3,W4. At the beginning of the week, the operator has only to finalise the planned unit commitment for the week 1. For week 2 he/she can wait, until he/she gets the information about the LPX-prices of week 1. As the LPX-prices between the weeks are correlated, he/she will use this additional information to improve the forecasts for the coming

3 weeks. According to this underlying decision process we choose four stages to build up the stochastic optimisation tree. At each stage representative scenarios for high prices, low prices and medium prices are considered, so that a decision tree with 27 scenarios is reached. The deterministic model has been built up and the expected value of the mean model has been calculated. For different turbines with different minimum operation and shut down times and varying amounts of coal of the take-or-pay contract, the value of stochastic solution is computed (see **Table 3.1**). The results show that stochastic optimisation in that case is very beneficial for inflexible turbines.

Table 3.1: Value of stochastic solution for flexible turbine and inflexible turbine

Minimum operation time/ minimum shut down time	Amount of coal of take-or-pay contract	Expected Value of profit [1000 EUR]		Value of stochastic solution [1000 EUR]
		Stochastic Model	Expected Value Model	
MIN_OP = 8h / MIN_SHUT_DOWN = 8h		2602	2484	118
		4646	4196	450
	1000 GWh	8045	8044	1
MIN_OP = 1h / MIN_SHUT_DOWN = 1h		2604	2601	3
		4654	4643	11
	1000 GWh	8046	8046	0

The reason is that the planned unit commitment for those turbines cannot be changed as easily as for flexible turbines. For inflexible turbines wrong decisions cannot be revised as easily as for flexible turbines, so taking not into account that prices can be very volatile leads to a non optimal unit commitment plan.

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